Modeling of Melt Layer Erosion and Splashing during Plasma Instabilities

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Outline

- Classical Analysis of Kelvin-Helmholtz Instability
- Modelling of Kelvin-Helmholtz Instability
**Kelvin-Helmholtz Instability**

**Question:** can the K-H instability develop under these plasma flow conditions with finger-like projections that break off to form droplets?

If the K-H instability can develop, then

- what is a minimal wavelength and the speed of plasma flow?

- what is the effect of the depth of melt layer?

- what is the effect of a magnetic field and its direction with respect to plasma flow?

For Li:

- \( \rho_m \sim 0.512 \text{ g/cm}^3 \)
- \( h_m \sim 0.2 \text{ cm} \)

For W:

- \( \rho_m \sim 17.6 \text{ g/cm}^3 \)
- \( h_m \sim 0.04 \text{ cm} \)
Classical K-H Instability Analysis

Sketch of the parallel flow of plasma and melt streaming respectively with velocity $V_p$ and $V_m$, of densities $\rho_p$ and $\rho_m$ ($\rho_m \gg \rho_p$) and of interfacial tension $\gamma$. Gravity acts in the negative z-direction.

Surface perturbation of the form:

$z = z_0 e^{i(k_x x + k_y y + nt)}$

The curvature of the surface:

$K = \kappa^2 z_0 e^{i(k_x x + k_y y + nt)}$

with $\kappa = \sqrt{k_x^2 + k_y^2}$

The pressure at the interface:

$p_p = p_m + \gamma K$

$V_p \sim 10^6 \text{ cm/s}$

$\rho_p \sim 10^{-9} \text{ g/cm}^3$

$g \sim 981 \text{ cm/s}^2$

for W: $\gamma \sim 2300 \text{ dyn/cm}$

$\rho_m \sim 17.6 \text{ g/cm}^3$

for Li: $\gamma \sim 405 \text{ dyn/cm}$

$\rho_m \sim 0.512 \text{ g/cm}^3$
Classical K-H Instability Analysis

Classical dispersion relation:

\[ n = -k_x \left( \frac{\rho_m V_m + \rho_p V_p}{\rho_m + \rho_p} \right) \pm \left[ \left( g_k \frac{\rho_m - \rho_p}{\rho_m + \rho_p} + \frac{\kappa^3 \gamma}{\rho_m + \rho_p} \right) \tanh(\kappa h_m) - \frac{k_x^2 \rho_m \rho_p}{(\rho_m + \rho_p)^2} (V_m - V_p)^2 \right]^{1/2} \]

- phase velocity of waves
- stabilizing gravity for long waves
- stabilizing tension for short waves
- destabilizing inertia

Instability → \[ \frac{\kappa^2 \rho_m \rho_p}{(\rho_m + \rho_p)^2} (V_m - V_p)^2 > \left( g_k \frac{\rho_m - \rho_p}{\rho_m + \rho_p} + \frac{\kappa^3 \gamma}{\rho_m + \rho_p} \right) \tanh(\kappa h_m) \]

Minimize this inequality relative \( \kappa \) to find a cut-off wavenumber from the condition:

\[ \frac{\partial f(\kappa)}{\partial \kappa} \bigg|_{\kappa = \kappa_c} = 0 \Rightarrow \]

\[ \kappa_c h_m (F + \gamma \kappa_c^2) (\tanh^2(\kappa_c h_m) - 1) + (F - \gamma \kappa_c^2) \tanh(\kappa_c h_m) = 0 \]

\( F = g(\rho_m - \rho_p) \) - gravitational restoring force

Classical K-H Instability Analysis

The case of deep melt: 

\[ h_m \to \infty \Rightarrow \tanh(\kappa_c h_m) \to 1 \]

\[ F - \gamma \kappa_c^2 = 0 \Rightarrow \kappa_c = \sqrt{\frac{g(\rho_m - \rho_p)}{\gamma}} \]

Cut-off wavelength

\[ \lambda_c = 2\pi \sqrt{\frac{\gamma}{g(\rho_m - \rho_p)}} \]

Criterion for the velocity difference

\[ (V_m - V_p)^2 > \frac{4\pi \gamma (\rho_m + \rho_p)}{\lambda_c \rho_m \rho_p} \]

The case of finite-depth melt:

numerical solution of the Non-linear Equation is required to find a cut-off wavenumber and wavelength and to determine the minimal relative speed for generation of the K-H instability

Classical K-H Instability Analysis

Magnetic field in the direction of flow

\[
n = -k_x \left( \frac{\rho_m V_m + \rho_p V_p}{\rho_m + \rho_p} \right) \pm \left[ \left( g \kappa \frac{\rho_m - \rho_p}{\rho_m + \rho_p} + \frac{k^3 (\gamma + \gamma_H)}{\rho_m + \rho_p} \right) \tanh(\kappa h_m) - \frac{k_x^2 \rho_m \rho_p}{(\rho_m + \rho_p)^2} (V_m - V_p)^2 \right]^{1/2}
\]

- phase velocity of the waves
- stabilizing gravity for long waves
- stabilizing tension for short waves
- destabilizing inertia

\[
\gamma_H = \frac{\mu H^2 k_x^2}{2 \pi \kappa^3}
\]

- magnetic surface tension

| V_m - V_p | \leq \sqrt{\frac{\mu H^2 (\rho_m + \rho_p)}{2 \pi \rho_m \rho_p}}

- magnetic surface tension

➢ the K-H instability will be additionally suppressed by a magnetic field if the relative speed does not exceed the root-mean-square Alfven speed

Magnetic field transverse to the direction of flow

- the dispersion relation is not changed

➢ development of the K-H instability in the direction of the flow is unaffected by a magnetic field transverse to this flow direction

Classical K-H Instability Analysis

The case of a melt layer with infinite depth $h_m \rightarrow \infty$

- $V_p \sim 10^6 \text{ cm/s}$
- $\rho_p \sim 10^{-9} \text{ g/cm}^3$

for Li:
- $\lambda_c \sim 5.64 \text{ cm}$
- $V_c \sim 9.5 \cdot 10^5 \text{ cm/s}$

for W:
- $\lambda_c \sim 2.29 \text{ cm}$
- $V_c \sim 3.6 \cdot 10^6 \text{ cm/s}$

- Perturbations with wavelengths smaller than a cut-off $\sim 5.64 \text{ cm}$ for Li and $\sim 2.29 \text{ cm}$ for W will be stable due to suppression by surface tension.

- Waves can be created on the surface with a wavelength greater than a cut-off when the relative velocity exceeds $\sim 9.5 \cdot 10^5 \text{ cm/s}$ for Li and $\sim 3.6 \cdot 10^6 \text{ cm/s}$ for W.
Classical K-H Instability Analysis

The case of a melt layer with finite depth $h_m$ – Non-linear Equation is solved numerically to find a cut-off $\lambda_c$ for each $h_m$

- cut-off wavelength *increases* with decrease of the thickness of a melt layer; it diverges for small melt depth $\sim 1.5$ cm
- relative velocity *decreases* with decrease of the thickness of the melt depth $h_m$ of a melt layer
Summary

- Waves with wavelengths below cut-offs, ~5.6 cm for Li and ~2.3 cm for W, will be *suppressed* on the surface of a deep melt due to tension effects.

- Cut-off wavelength *increases* with decrease of the thickness of a melt layer.

- Magnetic field *transverse* to the direction of the melt flow has no the effect on development of the K-H instability.
Two-Fluid Computational Model

- fluids with different physical and thermodynamic properties (out of thermodynamic equilibrium)
- fluids are separated by sharp interface and co-exist at every point in space and time with certain volume fractions
- each fluid is governed by its own set of balance equations closed by its own equation of state
- pressure and velocity relaxation procedures are used to establish the mechanical equilibrium between fluids
- source terms can be included for dissipative processes and phase transitions; equations for the number density of bubbles, granules, etc. can be added
Two-Fluid Computational Model

Mass conservation: plasma and liquid phase

\[
\frac{\partial \alpha_g \rho_g}{\partial t} + \frac{\partial \alpha_g \rho_g u_g}{\partial x} + \frac{\partial \alpha_g \rho_g v_g}{\partial y} + \frac{\partial \alpha_g \rho_g w_g}{\partial z} = 0
\]

\[
\frac{\partial \alpha_l \rho_l}{\partial t} + \frac{\partial \alpha_l \rho_l u_l}{\partial x} + \frac{\partial \alpha_l \rho_l v_l}{\partial y} + \frac{\partial \alpha_l \rho_l w_l}{\partial z} = 0
\]

\( \alpha_g \) and \( \alpha_l \) - gas and liquid volume fractions

\( \alpha_g + \alpha_l = 1 \)
Two-Fluid Computational Model

Momentum conservation: plasma phase

\[
\begin{align*}
\frac{\partial \alpha_g \rho_g u_g}{\partial t} + \frac{\partial}{\partial x} \left( \alpha_g \rho_g u_g^2 + \alpha_g p_g \right) + \frac{\partial}{\partial y} \left( \alpha_g \rho_g u_g v_g \right) + \frac{\partial}{\partial z} \left( \alpha_g \rho_g u_g w_g \right) &= \\
&= P_I \frac{\partial \alpha_g}{\partial x} + \lambda \left( u_l - u_g \right) - \alpha_g \rho_g g
\end{align*}
\]

\[
\begin{align*}
\frac{\partial \alpha_g \rho_g v_g}{\partial t} + \frac{\partial}{\partial x} \left( \alpha_g \rho_g v_g u_g \right) + \frac{\partial}{\partial y} \left( \alpha_g \rho_g v_g^2 + \alpha_g p_g \right) + \frac{\partial}{\partial z} \left( \alpha_g \rho_g v_g w_g \right) &= \\
&= P_I \frac{\partial \alpha_g}{\partial y} + \lambda \left( v_l - v_g \right)
\end{align*}
\]

\[
\begin{align*}
\frac{\partial \alpha_g \rho_g w_g}{\partial t} + \frac{\partial}{\partial x} \left( \alpha_g \rho_g w_g u_g \right) + \frac{\partial}{\partial y} \left( \alpha_g \rho_g w_g v_g \right) + \frac{\partial}{\partial z} \left( \alpha_g \rho_g w_g^2 + \alpha_g p_g \right) &= \\
&= P_I \frac{\partial \alpha_g}{\partial z} + \lambda \left( w_l - w_g \right)
\end{align*}
\]
Momentum conservation: *liquid phase*

\[
\frac{\partial \alpha_l \rho_l u_l}{\partial t} + \frac{\partial}{\partial x} \left( \alpha_l \rho_l u_l^2 + \alpha_l p_l \right) + \frac{\partial}{\partial y} \left( \alpha_l \rho_l u_l v_l \right) + \frac{\partial}{\partial z} \left( \alpha_l \rho_l u_l w_l \right) =
\]

\[
= -P_l \frac{\partial \alpha_g}{\partial x} - \lambda(u_l - u_g) - \alpha_l \rho_l g
\]

\[
\frac{\partial \alpha_l \rho_l v_l}{\partial t} + \frac{\partial}{\partial x} \left( \alpha_l \rho_l v_l u_l \right) + \frac{\partial}{\partial y} \left( \alpha_l \rho_l v_l^2 + \alpha_l p_l \right) + \frac{\partial}{\partial z} \left( \alpha_l \rho_l v_l w_l \right) =
\]

\[
= -P_l \frac{\partial \alpha_g}{\partial y} - \lambda(v_l - v_g)
\]

\[
\frac{\partial \alpha_l \rho_l w_l}{\partial t} + \frac{\partial}{\partial x} \left( \alpha_l \rho_l w_l u_l \right) + \frac{\partial}{\partial y} \left( \alpha_l \rho_l w_l v_l \right) + \frac{\partial}{\partial z} \left( \alpha_l \rho_l w_l^2 + \alpha_l p_l \right) =
\]

\[
= -P_l \frac{\partial \alpha_g}{\partial z} - \lambda(w_l - w_g)
\]
Two-Fluid Computational Model

Energy conservation: *plasma and liquid phase*

\[
\frac{\partial \alpha_g E_g}{\partial t} + \frac{\partial}{\partial x}(u_g(\alpha_g E_g + \alpha_g p_g)) + \frac{\partial}{\partial y}(v_g(\alpha_g E_g + \alpha_g p_g)) + \frac{\partial}{\partial z}(w_g(\alpha_g E_g + \alpha_g p_g)) = \]

\[
= U_I P_l \frac{\partial \alpha_g}{\partial x} + V_I P_l \frac{\partial \alpha_g}{\partial y} + W_I P_l \frac{\partial \alpha_g}{\partial z} + \mu P_l (p_l + \sigma \kappa - p_g) + \]

\[
+ \lambda U_I (u_l - u_g) + \lambda V_I (v_l - v_g) + \lambda W_I (w_l - w_g) - \alpha_g \rho_g u_g g
\]

\[
\frac{\partial \alpha_l E_l}{\partial t} + \frac{\partial}{\partial x}(u_l(\alpha_l E_l + \alpha_l p_l)) + \frac{\partial}{\partial y}(v_l(\alpha_l E_l + \alpha_l p_l)) + \frac{\partial}{\partial z}(w_l(\alpha_l E_l + \alpha_l p_l)) = \]

\[
= -U_I P_l \frac{\partial \alpha_l}{\partial x} - V_I P_l \frac{\partial \alpha_l}{\partial y} - W_I P_l \frac{\partial \alpha_l}{\partial z} - \mu P_l (p_l + \sigma \kappa - p_g) -
\]

\[
- \lambda U_I (u_l - u_g) - \lambda V_I (v_l - v_g) - \lambda W_I (w_l - w_g) - \alpha_l \rho_l u_l g
\]

\(\sigma\) – surface tension coefficient; for tungsten \(\sigma = 2300\) dyn/cm

\(\kappa = -\nabla \cdot \left( \frac{\nabla \alpha_l}{|\nabla \alpha_l|} \right)\) - interface curvature
Two-Fluid Computational Model

Volume fraction equation

\[
\frac{\partial \alpha_g}{\partial t} + U_I \frac{\partial \alpha_g}{\partial x} + V_I \frac{\partial \alpha_g}{\partial y} + W_I \frac{\partial \alpha_g}{\partial z} = -\mu(p_l + \sigma\kappa - p_g)
\]

Stiffened equations of state

\[
\rho_g e_g = \frac{(p_g + \gamma_g p_{\infty,g})}{(\gamma_g - 1)}; \quad \rho_l e_l = \frac{(p_l + \gamma_l p_{\infty,l})}{(\gamma_l - 1)}.
\]

\[
E_g = \frac{1}{2} \rho_g (u_g^2 + v_g^2 + w_g^2) + \rho_g e_g; \quad E_l = \frac{1}{2} \rho_l (u_l^2 + v_l^2 + w_l^2) + \rho_l e_l.
\]

Interface pressure and velocities

\[
P_I = \alpha_g \rho_g + \alpha_l \rho_l; \quad U_I = \frac{\alpha_g \rho_g u_g + \alpha_l \rho_l u_l}{\alpha_g \rho_g + \alpha_l \rho_l},
\]

\[
V_I = \frac{\alpha_g \rho_g v_g + \alpha_l \rho_l v_l}{\alpha_g \rho_g + \alpha_l \rho_l}; \quad W_I = \frac{\alpha_g \rho_g w_g + \alpha_l \rho_l w_l}{\alpha_g \rho_g + \alpha_l \rho_l}.
\]

for tungsten:

\[
\gamma_l = 2.2 \quad \text{and} \quad p_{\infty,l} = 1.41 \text{ Mbar}
\]
Two-Fluid Computational Model

A two-step numerical approach is used to solve the system of eleven equations:

**At step 1**

- eleven two-phase flow equations are solved using the MUSCL-TVDLF hyperbolic solver
- second order MUSCL-TVDLF numerical scheme was elaborated, further developed and applied for the first time to the 3D system of two-fluid flows
- new feature – non-conservative volume fraction equation is solved simultaneously with the balance equations

**At step 2**

- instantaneous pressure and/(or if needed) velocity relaxation is performed to restore the equilibrium of the two fluids
2D Air-Helium Kelvin-Helmholtz Instability
roll up of initial horizontal air-helium interface

- broken vortex cores and development of spikes near the interface - variations in air density is necessary condition for K-H
- small vortices and broken droplets dominate in the late stages
- pinch-off of the interface with formation of droplets
Plasma-Liquid Tungsten Instability

Plasma-liquid interface with random initial perturbation
Plasma density: \( \sim 0.01 \text{ g/cm}^3 \)
Plasma speed: \( \sim 10^6 \text{ cm/s} \)

- disruption of the interface within the melt depth \( \sim 1 \text{ cm} \)
- formation of liquid plumes, fingers and droplets drugged by the plasma flow

- topological structure of liquid patterns is highly irregular – no periodic array of compact spanwise K-H rollers
- velocity of liquid metal motion is \( \sim 2-5 \text{ m/s} \) deeply inside the melt layer; the velocity of melt fragments reaches up to \( \sim 150 \text{ m/s} \) at the surface
Plasma-Liquid Tungsten Instability

1. **Kelvin–Helmholtz instability mechanism: not observed**

- surface waves amplify forming finger-like projections that break off to form droplets
- depth of the melt affected is of the order of the wavelength of the surface disturbance

2. **Plasma-driven flow instability mechanism: observed**

- large droplets can be blown out by shear forces acting on the bulk of the molten metal
- impulse of the plasma flow can cause bulk fragmentation of the melt layer with ejection of large particle fragments
Conclusions

- high-speed ($\sim 10^4$-$10^5$ m/s) and dense (>0.01 g/cm$^3$) plasma flows over the liquid surface can generate the ejection of droplets from a homogeneous melt layer due to bulk shear forces.

- Introduction of bubbles and density inhomogeneities into a melted layer can significantly change its behavior and cause ejection of droplets for lower plasma densities and speeds.

- For typical ELM parameters, these predictions mean no melt splashing and droplet ejection from melt surface due to the K-H instability induced by plasma flow; JxB force could be the main mechanism.